DOCUMENT RESUME

ED 398 072 SE 058 758

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TITLE Arriving at the Pythagorean Theorem.
PUB DATE 95
NOTE 13p.
PUB TYPE Guides - Classroom Use - Teaching Guides (For Teacher) (052)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Concept Formation; Constructivism (Learning); *Geometry; High Schools; Lesson Plans
IDENTIFIERS *Pythagorean Theorem

ABSTRACT

This lesson plan uses group activity and manipulative materials to teach English-speaking students (ages 15-16) of diverse ethnic backgrounds an operatonal understanding of the Pythagorean Theorem. It is based on theories of constructivism and holism and includes teacher instructions, discussion questions, a retrospective vision, and an ancillary problem. (MKR)

Arriving at the Pythagorean Theorem

by

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ARRIVING AT THE PYTHAGOREAN THEOREM

by James A. Jaramillo, MA and Jonathan Caius Brown, MA

POPULATION: English speaking students of diverse ethnic backgrounds.

GRADE: Generally ninth graders.

AMBIANCE: Group activity of four students per table with team teaching.

THEORY: Constructivism and Holism.

TEACHING TECHNIQUES: Collective learning.

AGE: 15-16.

PRINCIPLES: Student use of closure, deduction, induction, and conservation.[A]

ABILITY:

An understanding of Van Hiele's (1986) first, second, and third levels of development in geometry.

1: Recognition of geometric figures (squares, etc.) via their appearance. Students can identify, name, and compare shapes.

2: Analysis of Properties: Students can describe the properties of various figures.

3: Informal deduction: Student can understand the role of definitions, relationship between figures; can order figures via their characteristics, and deduce facts logically from previously accepted facts utilizing informal arguments.

[A] These principles apply across domains and were developed by whole language theorists who seek to account for all aspects of information processing. *Closure* involves assigning labels to an object in a domain, and assigning properties to elements of the domain. *Deduction* is the use of some domain specific relation between labels that leads to another accepted one. *Induction* is the use of relations between labels to reach a new undefined relation. *Conservation* occurs when one creates the possibility of a generalized truth relation that exists and accounts for the new truth relation that was created.

GOAL:

By the end of this activity, students, using manipulatives and visual aids, should have developed an operational sense of the Pythagorean Theorem. In other words, a student should be able to engage in activities that overtly exemplify the relationship between deduction and induction.

PREREQUISITES: Students need to have acquired a working knowledge of identifying, naming, and comparing shapes; describing parts and properties of figures, and deductively deducing facts from previous facts via relationships between figures.[B] Secondly, students should know how to apply the area formula to shapes.[C]

MATERIALS: Each pair receives plastic figures of the following shapes[1]:

1 triangle (3"x 4"x 5") {**Appendix** C}
2 smaller squares (3"x 3") (**Appendix** 1A)
1 larger squares (5"x 5") (**Appendix** B)

CONCEPT OF MATHEMATICS: Teach the students an operational understanding of the Pythagorean Theorem.

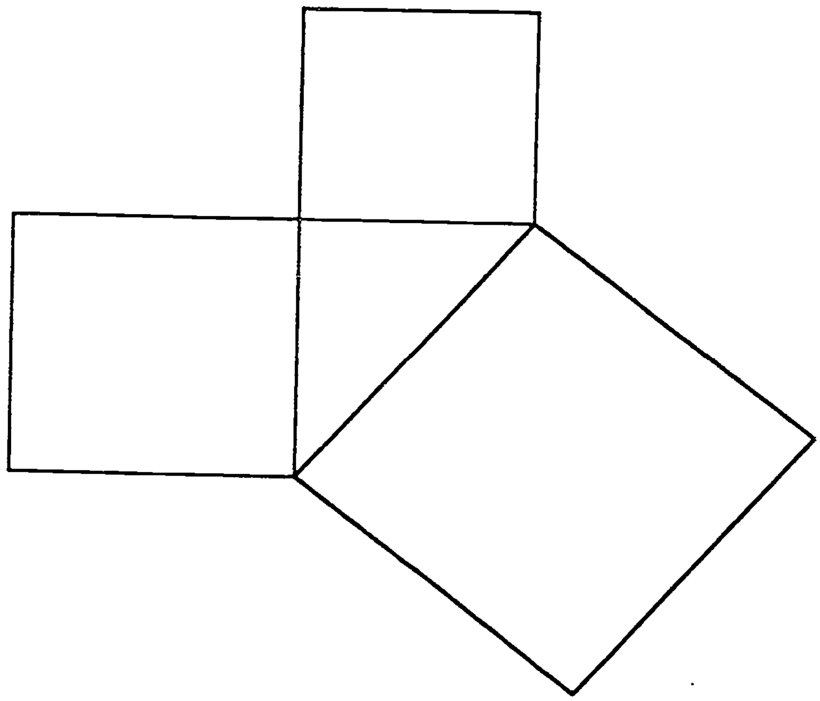
Using an overhead projector, teacher is to display the figure in the layout format as shown on **Figure 1**, and point to the shapes.

[B] Prior to employing this lesson plan, teachers need to know their students' ability, skill, and level of competence and, then, select lesson plans that meet the students' ability level and challenge them to advance (actual level of understanding). Thus, assessment provides a reference point about each student's competence level.

[C] This lesson plan is designed to cater to the spectrum of student geometry competence levels (i.e., egalitarian). Although some educators may devise a spectrum that consist of the following student levels: 1) closure, 2) the average exclusive deductive level, and 3) the inductive level, we assert that all students already employ these levels when they use language.

[1] If the instructor can provide tangible figures, then these preconceived figures are not necessary. However, we have enclosed them to visually enhance the instructor and student's comprehension of this lesson plan. Presumably, each student can cut out and mount these items to a tangible backing such as cardboard, milk carton lids, or construction paper to provide a firm shape.

FIGURE I



Answer the following questions:
1) What do you see?
2) Do you see any relationships between the figures?
3) Does the triangle have equal sides? **(No)**
4) What type of triangle is this? **(has a right triangle!)**
5) How do we know it is a right triangle? **(a 90 degree angle)**
6) Is there a side larger than the other two sides?
7) Which side is it?
8) What do we call this side? **(hypotenuse)**.

COMPREHENSION OF THE PROBLEM:

9) How do you measure area? **(Area=LxW)**
10) What is the area of the smaller square? **(3x3=9)**
11) What is the area of the middle square? **(4x4=16)**
12) What is the area of the largest square? **(5x5=25)**
13) What is the relationship between the sides of the triangle and the sides of the squares that they share?

INSTRUCTIONS:
14) Now, give the shapes to each pair of the group. (This act will subdivide the group of four students into pairs by colors). Teacher, now, instructs students to do the following:

15) Place the shapes in the pattern to resemble Figure 1's layout.

16) Measure the triangle's dimensions with a ruler, and indicate how many inches each side has by using a marker to write each respective measurement on the figure.

17) What are the measurements? **(3", 4", and 5")**

II. CONCEPT OF THE PLAN:

The teacher, now, asks students to record their measurements to reinforce their memory of the shapes dimensions.

Note: Figures are enclosed for two purposes: (1) to demonstrate how this lesson is to be carried out, and (2) function as one-dimensional facsimiles of three dimensional shapes for those teachers who do not have access to concrete shapes.

III. DIRECTIONS/EXECUTION OF THE PLAN:

18) Write each of your measurements in each box of the table on the next page.

19) What is the shortest length of the triangle?
 a) write your response under the letter A. (3)

20) What is the middle length?
 a) write your response under the letter B. (4)

21) What is the longest length?
 a) write your response under the letter C. (5)

22) Could each side of the triangle also be a side of the square that it shares? **(Yes)**

23) If students state no, ask students to look at the diagram and each side of the triangle again.

24) If yes, then ask, If we examine a square from each side of the triangle, what will be the measurement of the area of each square?
 (A=LxW)

25) What is the area of the smaller square?
 a) write your response under the second row of the letter A.
 (3x3=9)

26) What is the area of the middle square?
 b) Write your response under the second row of the letter B.
 (4x4=16)

27) What is the area of the largest square?
 c) Write your response under the second row of the letter C.
 (5x5=25)

TABLE I

A	B	C

28) Do you see a relationship between the smaller squares and the larger square?

If some students respond, "no," then ask them the following:

Does 9+16=25 or $A^2 + B^2 = C^2$?

29) If some students respond, "yes," then ask them, why?[2]

30) If students explain that the sum of the two smaller squares equals the area of the larger square, then ask them how did they determine this?

31) Since the smaller squares joined together are equivalent to the larger square and the area of the smaller squares is equivalent to each side times itself; and the area of the larger square is the larger side of the triangle by itself (hypotenuse), what can we say about the relationship between the smaller sides of the triangle and the larger side of the triangle?

(Each smaller side times itself (squared) added together equals the larger side squared (times itself).

32) Can you think of a way to say the same thing about any right triangle which possesses smaller sides of A and B with a larger side of C?

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[2] If students respond to this query, then the instructor needs to take the time to ask each respondent how he/she solved the problem (this should include each problem solving step) in an open forum setting, so that everyone listens and learns a new problem solving process.

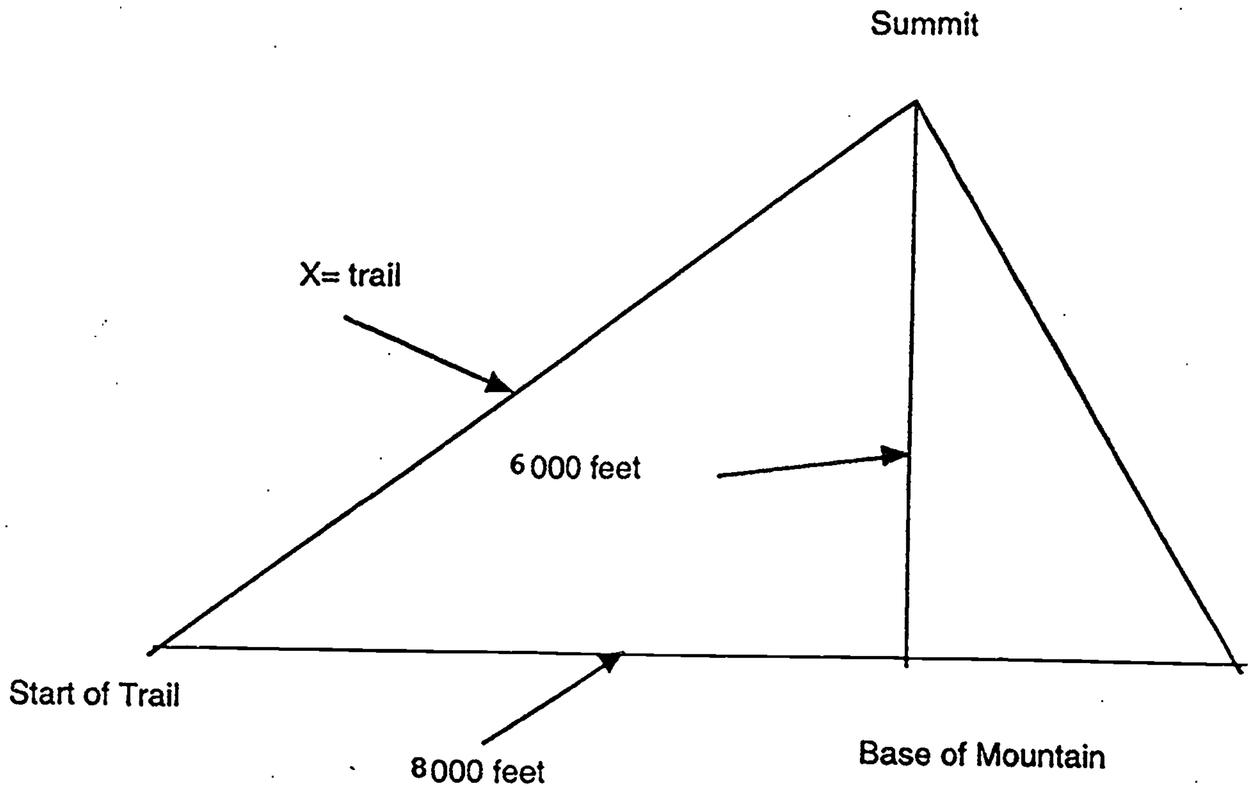
IV. RETROSPECTIVE VISION:

When students calculate and describe the results of their observations (i.e., measurement of triangle lengths and area of shapes) in an organized manner (i.e., Table 1), this indicates that they recognize geometric figures, describe their respective properties, and can deduce area and the relationship between figures This is based on their observations and the manipulatives they use to construct meaning from this activity. When students see the relationship between the smaller squares and larger squares, this demonstrates that they "inductively understand" this Euclidean geometry. If students accurately respond to question 32, this indicates that they understand the principle of conservativism.

Ancillary Problem

The next lesson is an extension of the previous one. Those students who master the first one are encouraged to solve the next story problem, which is designed to apply this abstract theorem to a real life situation. Based upon their knowledge of the theorem, they should be able to solve this realistic situation.

You are in the Valley of Phoenix and you are organizing a nature hike to the top of Superstition mountain. You know that the height of the mountain is 6,000 feet. The flat distance from the start of the trail to the route of the mountain is 8,000 feet. You only have a limited amount of basic supplies for this trip, and you need to ensure that everyone will complete the trip; therefore, you need to take the most direct route to the summit. And you need to know how long that distance is? (i.e., what's the length of the trail to the summit of the mountain?).



Lastly, if you desire so, design a word problem that demonstrates your knowledge of this theorem.

Appendix A

Appendix B

Appendix C